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Stability Derivatives of a Delta Wing with Straight Leading Edge in the Newtonian Limit

Asha Crasta*, S. A. Khan**
*(Research Scholar, Department of Mathematics, Jain University, Bangalore, Karnataka, India)
**(Principal, Mechanical Engineering Department, Bearys Institute of Technology, Innoli Mangalore, Karnataka, India)

ABSTRACT
This paper presents an analytical method to predict the aerodynamic stability derivatives of oscillating delta wings with straight leading edge. It uses the Ghosh similitude and the strip theory to obtain the expressions for stability derivatives in pitch and roll in the Newtonian limit. The present theory gives a quick and approximate method to estimate the stability derivatives which is very essential at the design stage. They are applicable for wings of arbitrary plan form shape at high angles of attack provided the shock wave is attached to the leading edge of the wing. The expressions derived for stability derivatives become exact in the Newtonian limit. The stiffness derivative and damping derivative in pitch and roll are dependent on the geometric parameter of the wing. It is found that stiffness derivative linearly varies with the pivot position. In the case of damping derivative since expressions for these derivatives are non-linear and the same is reflected in the results. Roll damping derivative also varies linearly with respect to the angle of attack. When the variation of roll damping derivative was considered, it is found it also, varies linearly with angle of attack for given sweep angle, but with increase in sweep angle there is continuous decrease in the magnitude of the roll damping derivative however, the values differ for different values in sweep angle and the same is reflected in the result when it was studied with respect to sweep angle. From the results it is found that one can arrive at the optimum value of the angle of attack sweep angle which will give the best performance.
Keywords: Straight leading edges, Newtonian Limit, Strip theory

I. INTRODUCTION

Unsteady supersonic/hypersonic aerodynamics has been studied extensively for moderate supersonic Mach number and hypersonic Mach number for small angles of attack only and hence there is evidently a need for a unified supersonic/hypersonic flow theory that is applicable for large as well as small angles of attack.

For two-dimensional flow, exact solutions were given by Carrier [1] and Hui [2] for the case of an oscillating wedge and by Hui [3] for an oscillating flat plate. They are valid uniformly for all supersonic Mach numbers and for arbitrary angles of attack or wedge angles, provided that the shock waves are attached to the leading edge of the body.

For an oscillating triangular wing in supersonic/hypersonic flow, the shock wave may be attached or detached from the leading edges, depending on the combination of flight Mach number, the angle of attack, the ratio of specific heats of the gas, and the swept-back angle of the wing. The attached shock case was studied by Liu and Hui [4] where as the detached shock case in hypersonic flow was studied by Hui and Hemdan both are valid for moderate angles of attack. Hui et.al [5] applied the strip theory to study the problem of stability of an oscillating flat plate wing of arbitrary plan form shape placed at a certain mean angle of attack in a supersonic/hypersonic stream. For a given wing plan form at a given angle of attack, the accuracy of the strip theory in approximating the actual three-dimensional flow around the wing is expected to increase with increasing flight Mach Number. The strip theory becomes exact in the Newtonian limit since the Newtonian flow, in which fluid particles do not interact with each other is truly two-dimensional locally. In this paper the Ghosh theory [6] is been used and the relations have been obtained for the stability derivatives in pitch and roll in the Newtonian limit.

II. Analysis

Consider a wing with a straight leading edge.
The Stiffness Derivative is given by

$$-C_{m_\alpha} = \sin \alpha_0 \cos \alpha_0 f(S_1)(\frac{2}{3} - h) \tag{1}$$

Damping derivative in pitch is given by

$$-C_{m_q} = \sin \alpha_0 f(S_1)(h^2 - \frac{4}{3}h + \frac{1}{2}) \tag{2}$$

Where $f(S_1) = \dfrac{\gamma+1}{2S_1}[2s_1 + (B+2S_1{}^2)/(B+S_1{}^2)^{\frac{1}{2}}]$

$S_1 = M_\infty \sin \alpha_O$

$B = (\dfrac{4}{\gamma+1})^2$ in all above cases.

In the Newtonian limit M_∞ tends to infinity and γ tends to unity. In the above expression (1) and (2) only $f(S_1)$ contains M_∞ and γ.

$$\lim_{M_\infty \to \infty} f(S_1) = \lim_{M_\infty \to \infty} \dfrac{(\gamma+1)}{2S_1}\{2S_1 + \dfrac{(B+2S_1{}^2)}{(B+S_1{}^2)^{\frac{1}{2}}}\}$$

$$= \lim_{S_1 \to \infty}\left[2 + \dfrac{(4+2S_1{}^2)}{S_1(4+s_1{}^2)^{\frac{1}{2}}}\right] = 4 \tag{3}$$

Therefore in the Newtonian limit, stiffness derivative in pitch,

$$\dfrac{-C_{m_\alpha}}{\sin 2\alpha_0} = 2(\dfrac{2}{3}-h) \tag{4}$$

The Damping derivative in Newtonian limit for a full sine wave is given by

$$-C_{m_q} = 4\sin\alpha_0(h^2 - \dfrac{4}{3}h + \dfrac{1}{2}) \tag{5}$$

We define $g(h) = (h^2 - \dfrac{4}{3}h + \dfrac{1}{2})$ which is a quadratic in pivot position h and hence has a minimum value

$$-C_{m_q} = 4\sin\alpha_o g(h)$$

In Eq. (5), only g (h) depends on h and other terms are constant. To get minimum value of C_{m_q} only g (h) is to be differentiated and putting $\dfrac{\partial}{\partial h}g(h)$ equal to zero

$$\dfrac{\partial}{\partial h}[(h^2 - \dfrac{4}{3}h + \dfrac{1}{2})] = 0$$

$$\therefore h = \dfrac{2}{3}$$

Let the value for h corresponding to $[C_{m_q}]_{\min}$ be denoted h_m.

$$\therefore h_m = \dfrac{2}{3}$$

Hence $g(h)_{\min} = (h^2{}_m - \dfrac{4}{3}h_m + \dfrac{1}{2})$

$$\therefore -C_{m_q} = 4\sin\alpha_o g(h)_{\min}$$

$$\therefore \dfrac{-C_{m_q \min}}{\sin\alpha_o} = 4g(h)_{\min}$$

(6)

Rolling Moment due to roll in Newtonian limit becomes

$$-C_{l_p} = \dfrac{4\sin\alpha_0 \cot\in}{12}$$

$$\therefore \dfrac{-C_{l_p}}{\sin\alpha_0} = \dfrac{\cot\varepsilon}{3}$$

(7)

RESULTS AND DISCUSSION

The main purpose of this study is to find stiffness and damping derivatives in pitch of a delta wing with straight leading edge in the Newtonian limit where the Mach number will tend to infinity and the specific heat ratio gamma will tend to one, in this situation when the stability derivatives are considered how they vary with the change in the geometric parameters.

Fig. 1 shows the variation of stiffness derivatives with the non-dimensional pivot position. From the figure it is seen that at the leading edge it has the maximum value of around 1.35 and the downstream of the leading edge it decreases with increase in the pivot position where as the minimum value at the end of the trailing edge is around -0.75. It is also seen that the location of center of pressure has also, shifted towards the trailing edge of the wing otherwise this non-dimensional pivot location is in the range from 50 to 60 percent from the leading edge, when they are evaluated for supersonic and hypersonic Mach numbers, this shift towards the trailing edge will be advantageous from the static stability point of view as with this shift of center of pressure will result in higher values of the static margin and hence large value of the stiffness derivative. When the damping derivatives are considered it is found that there is nonlinearity in the damping derivative with respect to the pivot position in Newtonian limit as shown in Fig. 2. From the figure it is seen that at the leading the magnitude of the damping derivative is 2 and at the trailing edge is around 0.75. If we compare these results of the damping derivatives with that for low supersonic and hypersonic Mach numbers, it is found that the initial

value itself is increased by 30 percent that it continues to be high value for all the values of the non-dimensional pivot position. Also, it is seen that the minima of the pitch damping derivative too has shifted towards the trailing edge resulting in the higher values of the pitch damping derivative and hence will perform better in the dynamic conditions as compared to those at supersonic and hypersonic Mach number. The physical reasons for this trend may be due to the change in the pressure distribution on the wing plan form. Results of stiffness derivative as a function of angle of attack are shown in Fig. 3, for a fixed pivot position of   h = 0 and h = 0.6. From the figure it is seen that the Stiffness increases linearly with angle of incidence. It is also seen that stiffness derivative assumes very high value when it is computed at the leading edge and the center of pressure and these assumes importance while analyzing the performance of the wing, as stiffness derivative is a measure of static stability. At the design stage depending upon the requirement we need to work the balance between the upper and lower limit of the static margin.

Fig. 4 shows the variation of the damping derivatives with respect to angle of attack for three pivot positions namely h = 0, 0.6, and 1.0, this was considered just ascertain the capability of the wing under three different situations. It is well known that stiffness derivative ensures the static stability of the wing, but if the system is statically stable that doesn't mean that it is dynamically stable as well; hence damping derivative assumes a very important role in the analysis of the stability and control. From the figure it is also, seen that damping derivative varies linearly for all the values of the pivot position. The reasons for this behavior may due the computational domain as they are computed at the locations of the wing leading edge, the center of pressure, and at the trailing edge; where the location has been fixed and the effect of the angle of incidence alone is being reflected in these results.

Results for roll damping derivative as a function of angle of incidence are shown in Figs. 5 to 7. From the figure it is seen that roll damping derivatives, also varies linearly with respect to the angle of attack. When the variation of roll damping derivative was considered, it is found that it also, varies linearly with angle of attack for given sweep angle, but with increase in sweep angle there is continuous decrease in the magnitude of the roll damping derivative however, the values differ for different values in sweep angle and the same is reflected in the result when it was studied with respect to sweep angle.

Fig. 5 shows results for roll damping derivative variation with angle of incidence for sweep angle in the range five degrees to thirty degrees. It is

found that the difference in the values of the roll damping derivative is very high between sweep angles from five to 10 degrees, then this decrease in the magnitude is gradual. The reason for this behavior may be due to the presence of the ratio of cosine and sine term; and for zero sweep angle the value of cosine term is one, whereas sine term is zero, and this value is getting reversed when sweep angle is ninety degrees. The similar trends are reflected in Figs. 6 and 7 when sweep angle in the range from thirty five to eighty degrees.

Figs. 8 and 9 present the results of roll damping derivative as a function of angle of attack. It is seen that when angle of attack is in the range from five degrees to fifteen degrees optimum sweep angle may be in the range ten degrees to twenty degrees as seen in figure 8, where if the angle of attack range is between twenty to thirty degrees then sweep angle in the range from fifteen degrees to fifty degrees may the best option but one has to analyze the further, keeping in mind that whether; for the given combination, the leading edge of the wing will be sub-sonic or supersonic. Hence, one has to consider all these parameters to arrive at the optimum solution.

III. Conclusion

The present theory is valid when the shock wave is attached to the leading edge. The effect of secondary wave reflections and viscous effects are neglected. The expressions derived for stability derivatives become exact in the Newtonian limit. From the results it is found that the stability derivatives are independent of Mach number as they are estimated in the Newtonian limit, where Mach numbers will tend to infinity and specific heat ratio gamma will tend to unity. The stiffness derivative and damping derivative in pitch they reflect only the effect of the geometric parameter of the wing in the Newtonian limit too. It is found that stiffness derivative linearly varies with the pivot position as the same was found for the cases in our previous results at low supersonic, supersonic, and Hypersonic Mach numbers, however, the location of center of pressure has shifted towards the trailing edge by nearly twenty percent which; will result in increased value of the static margin and hence increase value of stiffness derivative resulting in better performance in the dynamic conditions. In the case of damping derivative since the expression for the damping derivative is non-linear and the same has been reflected in the results. For damping derivative also, the initial value also has increased followed by shifting of center of pressure towards the trailing edge resulting in better performance of the plane. Roll damping derivatives also vary linearly with angle of attack for a given value of sweep angle; however, for a given angle attack when roll damping

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derivatives were plotted with sweep angle the behavior is non-linear. From the above results it can be stated that if angle of attack range is between ten degrees to fifteen degrees then the optimum sweep angle will be in the range five to twenty degrees, however, if the angle of attack is in the range of twenty to thirty degrees then the optimum range of the sweep angle will be between fifteen to fifty degrees.
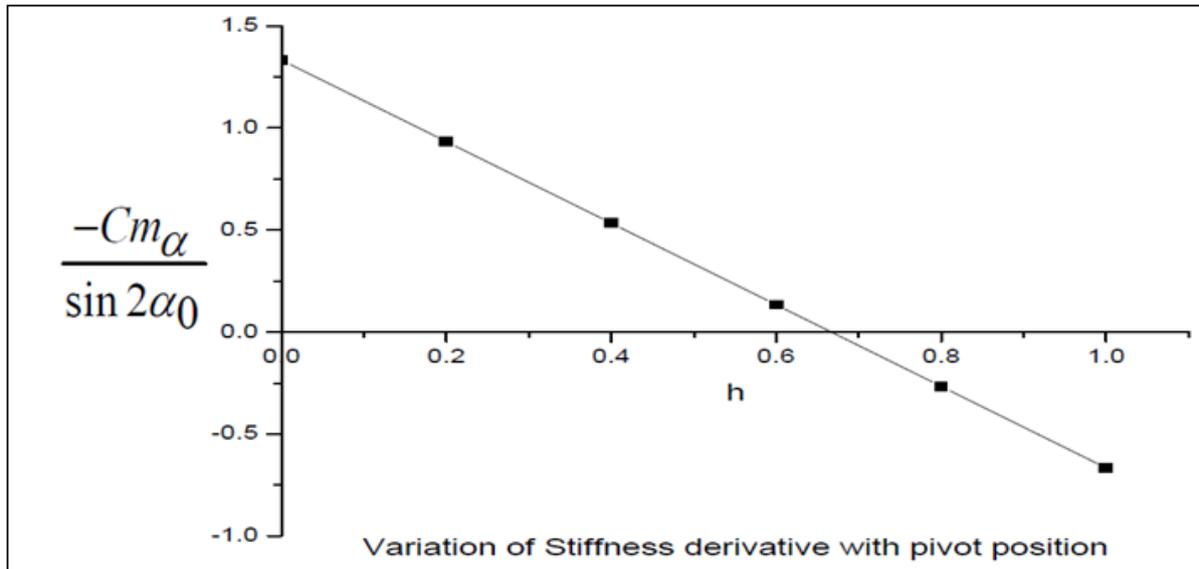
Fig. 1: variation of stiffness derivative in pitch with pivot position
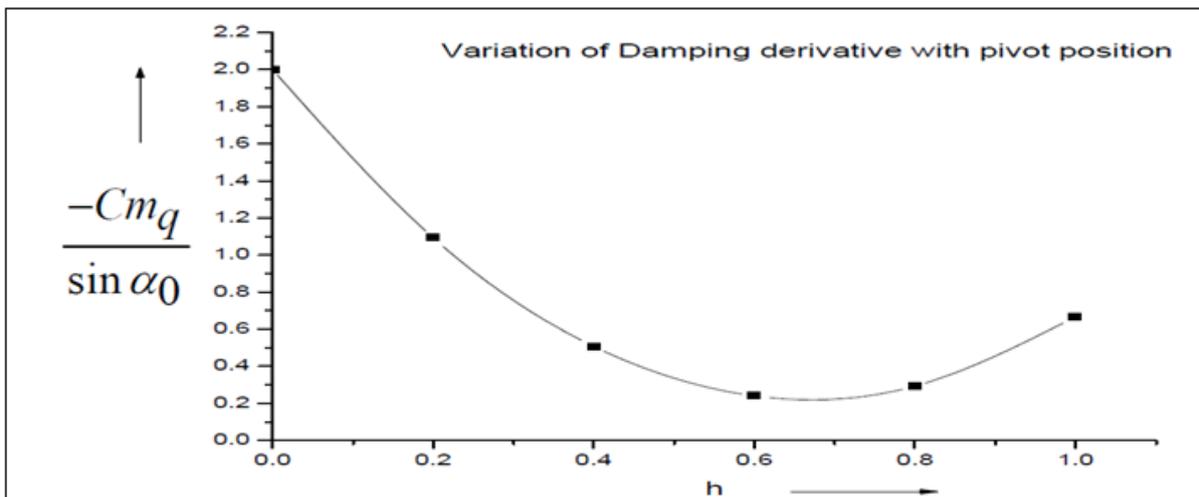
Fig. 2: variation of damping derivative in pitch with pivot position
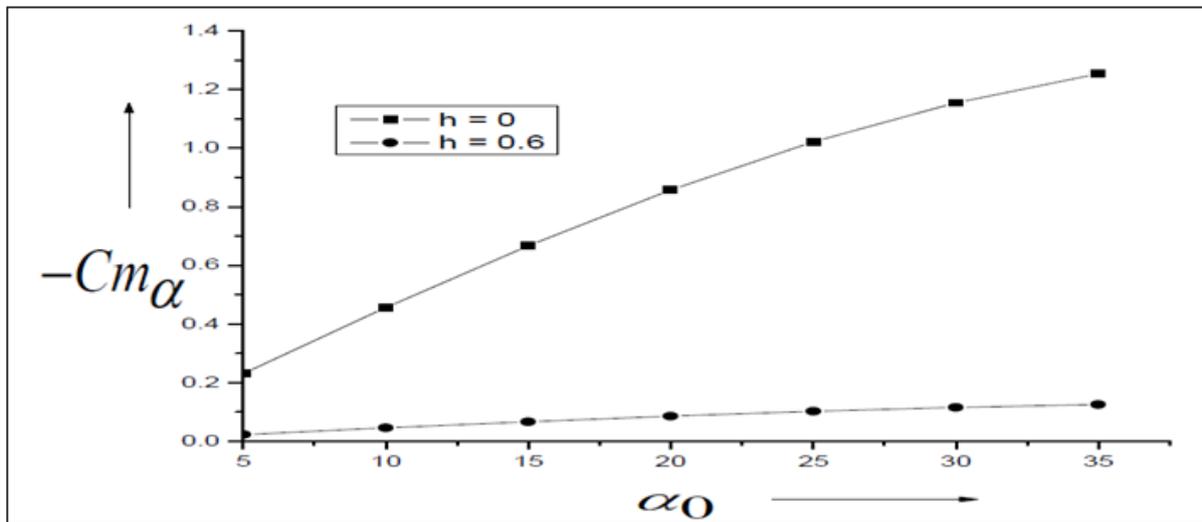
Fig. 3: Variation of damping derivative in pitch with angle of incidence
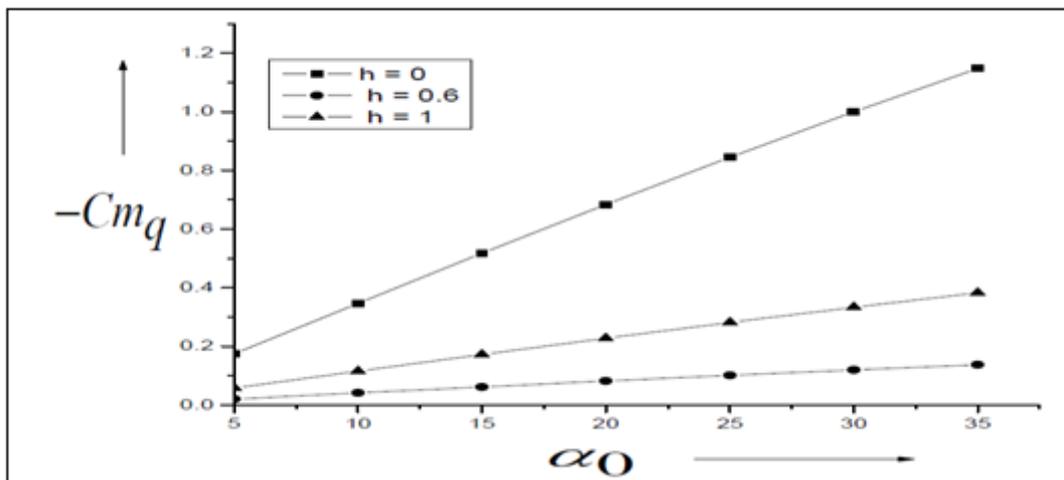
Fig. 4: variation of stiffness derivative in pitch with angle of incidence
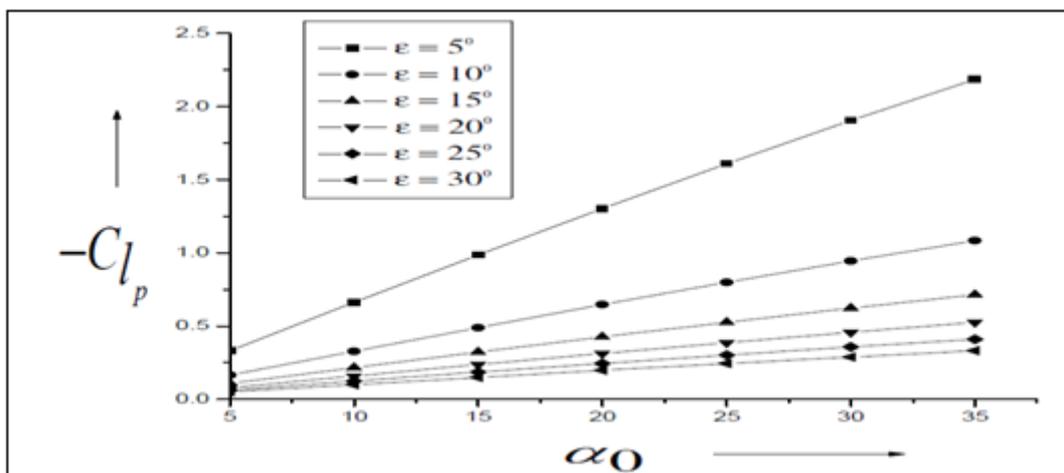
Fig. 5: variation of Roll damping derivative with angle of incidence
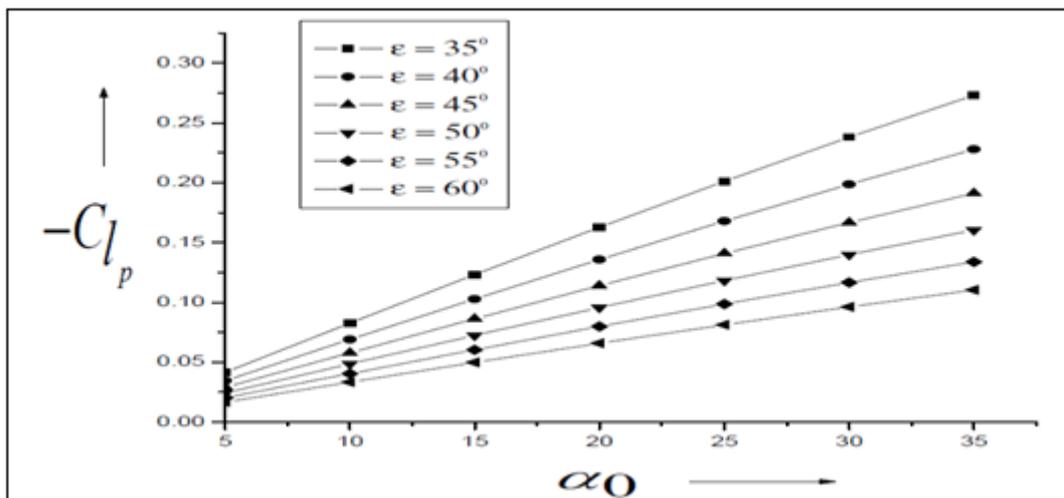
Fig. 6: variation of roll damping derivative with angle of incidence
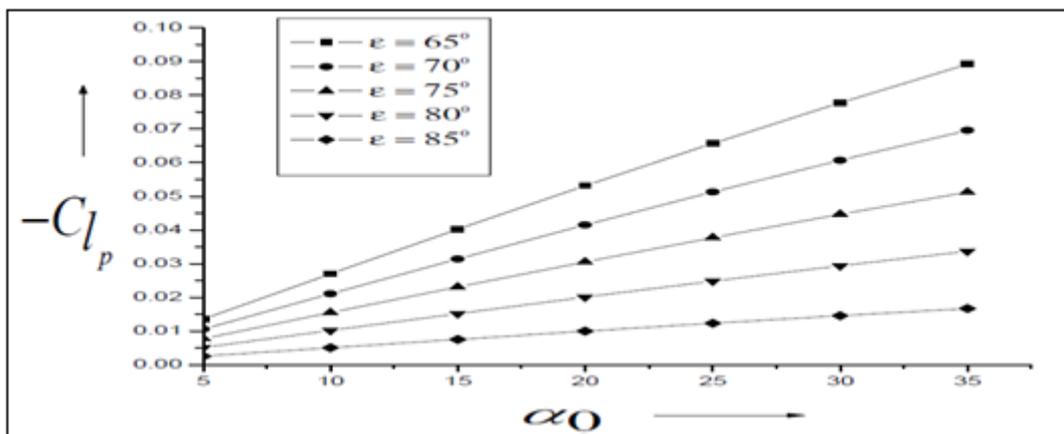
Fig. 7: variation of roll damping derivative with angle of incidence
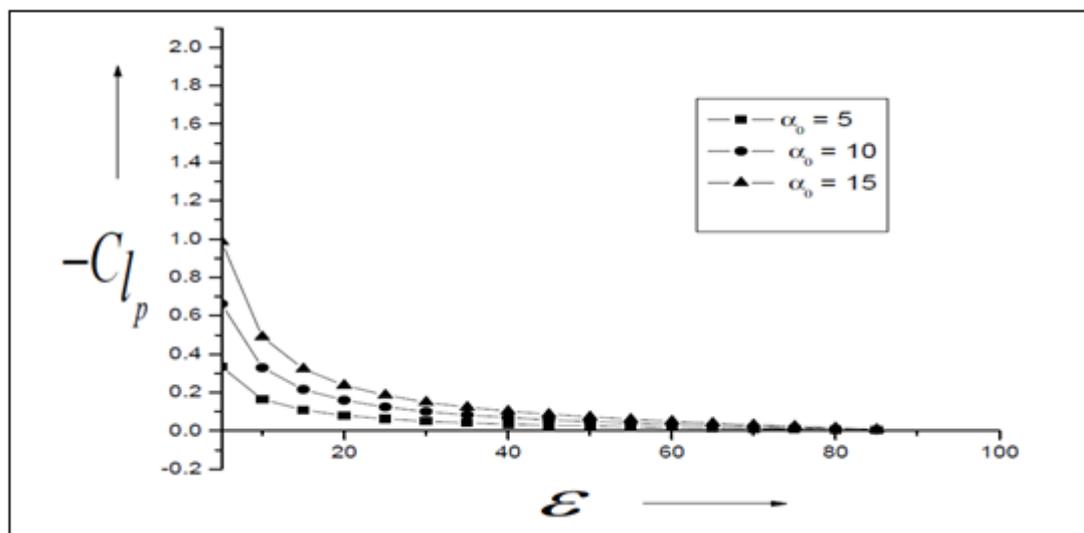
Fig. 8: variation of roll damping derivative with Sweep angle
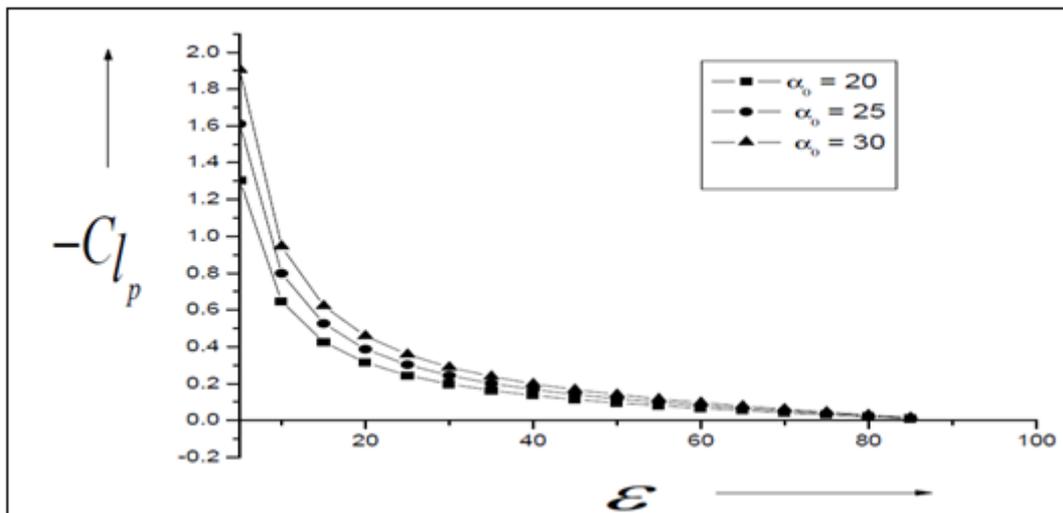
Fig. 9: variation of roll damping derivative with Sweep angle

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